namibia university
OF SCIENCE AND TECHNOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 08BHAM	**LEVEL:** 8
COURSE CODE: ADC801S	**COURSE NAME:** ADVANCED CALCULUS
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	DR ALFRED KAMUPINGENE
MODERATOR:	DR. D. MAKINDE

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.
4. Start answering each of questions 1, 2, 3,4, 5, 6 and 7 on a new page.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Question 1 (10 marks)

Prove that the function $f(x) = x^3 + 4x^2 - 7x - 9$ satisfies the conditions of Rolle's theorem in the interval $I = [-1, 2]$. Find the value of ξ such that $f'(\xi) = 0$. (10)

Question 2 (10 marks)

Consider the $f(x) = c_1 x^2 + c_2 x + c_3$ where $c_1 \neq 0$. Show that the number c in the conclusion of the Langrage mean value theorem is always the midpoint of the given interval $[a, b]$. (10)

Question 3 (16 marks)

Given a function $f(x)$ defined by a power series of the $f(x) = \sum_0^\infty c_n {}_{(x-a)^n}$ with a radius of convergence $R > 0$, determine the coefficients c_n in terms of the derivatives of the function. (16)

Question 4 (19 marks)

Compute the Maclaurin series of the function $f(x) = \dfrac{\sin(x^2)}{x^2}$ as well as the integral

$$\int_0^x \sin \frac{(s^2)}{s^2} ds$$ (19)

Question 5 (19 marks)

Determine the location and nature of the stationary points of the function $f(x, y) = x^4 - 2(x - y)^2 + y^4$ by using the Hessian of the Taylor polynomial. (19)

Question 6 (19 marks)

What is the maximum area of a rectangle enclosed by the x and y axes and the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1?$$ (19)

Question 7 (7 marks)

Given the vector $A = xz^3 i - 2x^2 yz j + 2yz^4 k$, find $\nabla x \cdot A$ at the point with coordinates $(1, -1, 1)$ (10)

END OF PAPER   **TOTAL MARKS: 100**